Cooperative learning and social constructivism in mathematics education

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This paper reports on a study which attempted to translate research on learning from a constructivist perspective into a teacher education classroom environment. The study implemented a social constructivist view of learning mathematics for the purpose of analysing the practice of teaching and learning mathematics. Learning opportunities that emerged from the students' experiences in the classroom environment not only helped to facilitate the students' own mathematical development but also provided insight into the role of the mathematics teacher in the classroom.

Research Perspective

It is now well accepted that, according to the constructivist view of learning mathematics, students construct their own mathematical knowledge rather than receiving it in finished form from the teacher or a textbook. Within this framework, this means students create their own internal representations of their interactions with the world and build their own networks of representations (Hiebert & Carpenter, 1992). However there is considerable countenance given in the literature to the view that constructivist perspectives of teaching are considerably less developed than are constructivist perspectives of learning (Simon, 1995) and that educators are faced with major difficulties when attempting to develop pedagogical implications from the constructivist ideas about learning (Prawat, 1992; Wood, Cobb & Yackel, 1993).

Our work in exploring ways to implement the constructivist paradigm has been influenced by Piaget's and von Glasersfeld's constructivist epistemology that emphasises the role of cognitive conflict, reflective abstraction, and conceptual reorganisation in mathematical learning (Piaget, 1980; von Glasersfeld, 1988). The key constructs around which our study is built are an experiential learning cycle adapted from Jones and Pfeiffer (1975) which uses principles of cooperative learning and the problem-centred approach of the Purdue Problem-Centered Mathematics Project (Wood, Cobb & Yackel, 1993). Our work has also been influenced by Vygotsky's (1978) analysis of the crucial role that social interaction plays in learning. Vygotsky has commented on the process whereby interpersonal language (the language used to appeal to others) becomes intrapersonal language (the guiding language of self actualisation) through relating to the consequences of behaviour, activity, norms, and attitudes. 'When children develop a method of behaviour for guiding themselves that had previously been used in relation to another person, when they organise their own activities according to a social form of behaviour, they succeed in applying the social attitude to themselves' (Cam, 1995, p.9). Vygotsky's work implies that learners who experience the processes involved in thinking together will come to experience the self actualisation of the processes involved in their own thinking. Similar themes emerge from the work of George Herbert Mead (1934) who claims that human beings would never have developed understandings without the capacity to talk to themselves; and that true understanding develops from our communion with others.

If learning from social and interactive experiences is important, as social constructivists claim, then we need to change our approach to teaching mathematics and teacher education. Constructivism requires of us that we provide our student teachers with appropriate forms of experiences. In keeping with such views our study was established with the belief that mathematical learning is just as much an interactive as a constructive activity (Cobb, 1990). We endeavoured to organise a classroom which afforded opportunities for students to interactively constitute their understanding through interpersonal and intrapersonal communication, small and large group discussion, and in a cooperative environment where the teacher does not provide answers but more importantly encourages and mediates discussion. This is similar to the teaching perspective developed by Kuhs and Ball (1986) in that 'the teacher aids the student by questioning, challenging, and offering experiences that reveal the inadequacy of inappropriate conceptions' (p.5) but refrains from 'dishing out answers'.

The approach taken in this study sought to include aspects of the *National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991, pp.16-20):

* learners construct their own meanings from, and for, the ideas, objects and events which they experience;
* learning happens when existing conceptions are challenged;
* learning requires action and reflection on the part of the learner;
* learning involves taking risks;
* mathematics learning is likely to be enhanced by feedback;
* mathematics learning is likely to be enhanced by using and developing appropriate language;
* mathematics learning is likely to be enhanced by challenge within a supportive framework.

According to the *Standards* (NCTM, 1989) 'Instruction [in mathematics education] has emphasised computational facility at the expense of a broad, integrated view of mathematics and has reflected neither the vitality of the subject nor the characteristics of the student' (p.65). Much research suggests that students are too passive and need to become more involved intellectually in classroom activities (cf. Goodlad, 1983). Still, too many mathematics lessons require students to do little more than listen passively. Some educators believe that cooperative learning can be a useful strategy for responding to more meaningful experiences which increase coherence and understanding in the learning of mathematics.

Research Emphases

Our research objective was to analyse the mathematical learning of student teachers in a classroom where instruction was broadly compatible with social constructivism. The primary aim of the study was to measure students' levels of mathematical achievement after a semester in an elective subject called *Mathematics for K-6 Teachers*. Our classroom approach used paired groups to encourage discussion as students' sought to solve mathematical problems. After exploring problems in pairs, whole-class student-led sharing sessions allowed students to further extend their collaborative efforts to construct meaning by clarifying or verifying solutions. The content of the classes came mainly from the NSW HSC syllabus *Mathematics in Society* and the problems used are similar to those in past examination papers.

The time for learning was one semester (unlike the general length of time for the HSC subject of two years) and the onus of finding time to come to grips with the topics was on the learner. The classroom environment was specifically developed to show students an approach which is an alternative to ones commonly experienced during their high school education. The key feature of this approach is the use of an experiential learning cycle which incorporates the following stations:

1 Experiencing. Students must be actively involved in their own learning. They must engage in

activities which engender their mathematical thinking. These activities may involve physical action with materials but will involve mental action. Learning must involve 'doing' in order to be effective.

2 Discussing. Reactions and observations arising from the experiences need to be shared with fellow learners and other members of the community and talked about in order for them to be evaluated and, perhaps, validated against the taken-as-shared knowledge of the learner's community. Explanation, justification and negotiation of meaning through communication will help the learner establish this knowledge.

3 Generalising. Learners need to develop for themselves, through individual construction and interaction with their communities, hypotheses which indicate the current state of their understanding. These hypotheses, or generalisations, will then be tested for viability through their application to other problematic situations or further communicative discourse. It is these generalisations which form the basis for the learner's next experience.

4 Applying. Planning how to use the new or revised learning and actually applying it to other contextual situations will not only validate it as viable knowledge (or suggest rejection of it as non-viable) but will also provide the learner with another experience which could be used to commence yet another cycle.

An essential feature of the classroom environment was the interactive constitution of a set of social norms to provide a framework for (co)operating. The following norms were developed by one class:

1 Activities will consist of problems for the students. That is, it is assumed that the students may not be able to obtain solutions or even to know where to start immediately.

2 When working in small groups, students are expected to develop solutions to the activities cooperatively and to reach consensus on these solutions. The teacher is expected to circulate among the groups, observing their interactions and encouraging their problem-solving attempts.

3 Students are expected, as a small group, to explain and defend their solutions or attempts at solutions to the whole class. Other students are expected to indicate their agreement or disagreement and to encourage alternative solutions.

4 The whole class is expected to see itself as a community of validators and is expected to work towards a solution or solutions which can be taken-as-shared. It is not the teacher's role to validate solutions.

Data Collection

Each student was expected to record, in a journal, their reactions to the course, attempts at solutions to the activities, and any other feelings or concerns they may have had. The students were encouraged to make and record summaries of their discussions and generalisations that developed during and between classes. Students' written records in their journals, video recordings of their participation in classes, audio recordings of final-week interviews and final examinations provided case study data on how mathematical thinking and attitudes changed. There were also attitudinal entry and exit results for the 37 students who participated. Data came from four different classes taken by two different teachers and in two different modes (that is, two classes attended weekly for three hours for 13 weeks while two classes attended the same number of classes but in three blocks of two weeks spread over 13 weeks). Class sizes varied from 8 to 16.

In the process of undertaking analysis of the data (Geoghegan et al., 1994) we became aware of the complexity and significance of various dimensions of interaction and cooperation that were impacting upon students' learning. By reflecting upon traditional misconceptions of classroom interaction, we realised how the project had illustrated the nature of cooperative learning in the classroom. The following section of this report is an attempt to demystify three prevailing myths about cooperative learning by discussing episodes of the students' participation in our study.

Discussion

Myth 1. There is a belief that in cooperative learning situations the student who messes about, or the student who won't work, or is not interested, will rely on the capable students in the group to do all the work.

Student 1 was inclined to be disruptive and disinterested in conforming to the class norms in the early sessions. In an apparent attempt to disrupt proceedings, through his seemingly precocious nature, he spasmodically challenged and provoked students as they presented their solutions to the class. However his disinclination to participate (he was often seen asleep) and his apparent disinterest gradually gave way to more concerted efforts to comply with the class norms and participate more productively. This is attributed much to the expectation of individual accountability placed upon him to contribute, with his partner, with explanations of his solutions to problems in front of the whole class. It was important for him to develop a feeling of personal responsibility for the achievement of the group's goal. In time he came to appreciate the importance of his contribution to not only the group's success but also his own. Interestingly this student, midway through the semester, withdrew from all his subjects except this one in which he continued and became an enthusiastic and even more precocious participant.

Myth 2. There is a belief that bright students will be held back in cooperative learning situations.

Student 2 was an experienced HSC mathematics student and was sometimes noticed twiddling her thumbs while her partner agonised over a problem. On several occasions she confidently called out what she considered to be correct solutions to particular problems only to have her contribution disputed by a large proportion of the class. Challenged by the other students' reaction, she would immediately articulate her thinking and find, with prompting from the rest of the class, where she had made an error in her calculations. This kind of productive and constructive interaction was evidenced frequently and helps to illustrate the value of reflective thinking and metacognition as important aspects in the process of having to talk through, explain and clarify understandings. By explaining and clarifying their thinking students stand to enhance higher-level reasoning strategies which in turn can enhance learning ability (Cambourne & Turbill, 1987). Brighter mathematical thinkers might not have developed interpersonal skills in communicating, listening, being patient, being tolerant, sharing, developing friendships, and developing collaborative decision-making skills. Such skills are important problem-solving abilities and are not necessarily learnt from text books or part of being mathematically able.

An entry from a student's journal elaborates further:

Student 3: '[The] simple explanation made me think about other students in the class (my partner, and a few others in particular). Why couldn't they explain this to me when I asked them why they multiplied to reach the solution. Does this mean that like me (before this class) they relied on a formula without understanding why it worked or was it simply a case of them finding it difficult to put their thoughts into words. I really don't know, but one thing is for sure, this class has certainly given me a lot to think about lately. I'm finding out more and more about myself and the way I think mathematically, but perhaps that's exactly what this class is suppose to do. Is it?'

Myth 3. There is a belief that the teacher has control of all knowledge.

From a journal entry:

Student 4: 'As I had already begun working on activity 18 and I knew that other students were experiencing difficulties with these particular problems, I suggested that it might be a good idea if I could help these students by working with them. In the end I was really pleased that I did this, as [Students A and B] seemed to appreciate the fact that I was prepared to go over some of the problems that had caused them trouble in the previous tutorial. After explaining how and why a particular step was taken they both said they understand how the solution was reached. However to ensure that they were not just saying this, I made up a few other questions based on the questions given so I could actually see for myself whether they really understood or not; thankfully it turned out that they did.'

Student A (from the above episode wrote in her journal): '[Student 4] what a gem. She spent time with us today. She didn't treat us like we were the 'wombats' she was wonderful. I would be proud to have her teach my child. Anyway she did her best to explain trigonometry to us. She even used concrete materials. She had to explain a little of Algebra. This was good for me because ... I understood what she said.'

Student B (also from the above episode wrote): 'What a difference it made to have [Student 4] lend her assistance and bring the level to one that I could be comfortable with. Actually while I think of him, Bruner developed the relevant metaphor of "scaffolding". Bruner's notion that a more expert person, adult or older peer helps the less expert person through learning how to deal with a problem, initially providing a great deal of support, which is gradually reduced as the learner becomes able to take on the task more independently makes sense to me more than ever now.'

Such evidence indicated that many students in the program not only developed their mathematical knowledge but also their understandings of the teaching and learning process from a constructivist perspective.

Summary

The above episodes emphasise the value of allowing time, instilling confidence, encouraging communication, and collective ownership of solutions developed through classroom interaction. The cooperative learning that occurred in the classroom was a feature recognised by the students as important in their learning. A student remarked in the final interview:

Student 5: 'I specifically liked having to go in there and doing maths but not on my own. If on my own I always make mistakes - going over and over it again. But when I've got someone with me to talk to then it gets easier to get my answer quicker ... I actually sat down on the bus and got my pen and started working. I've not done that before. ... I just had to do it. I liked to get into it. It really got me in.'

And in her journal she commented: 'If I was to work alone I do not believe that I would have learnt anything at all except negative feelings about my abilities.'

Overall, considerable data indicated that cooperative learning was a clearly expressed support for our students in developing positive attitude changes towards learning and teaching mathematics; many students attributing much of their success in their mathematical development to the cooperative environment in which they (co)operated.

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